1987

# CALCULUS AB

## SECTION II

Time—1 hour and 30 minutes
Number of problems—6
Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination $\ln x$ denotes the natural logarithm of $x$ (that is, logarithm to the base $e$). (2) Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

1. A particle moves along the $x$-axis so that its acceleration at any time $t$ is given by $a(t) = 6t - 18$. At time $t = 0$ the velocity of the particle is $v(0) = 24$, and at time $t = 1$ its position is $x(1) = 20$.
   - (a) Write an expression for the velocity $v(t)$ of the particle at any time $t$.
   - (b) For what values of $t$ is the particle at rest?
   - (c) Write an expression for the position $x(t)$ of the particle at any time $t$.
   - (d) Find the total distance traveled by the particle from $t = 1$ to $t = 3$.

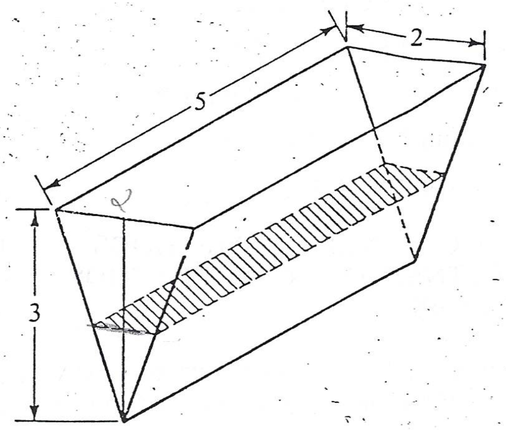
2. Let $f(x) = \sqrt{1 - \sin x}$.
   - (a) What is the domain of $f$? ??? $1 - \sin x \geq 0$ set of all real $x \neq \frac{\pi}{2} + 2n\pi$
   - (b) Find $f'(x)$.
   - (c) What is the domain of $f'$? ??? $1 - \sin x \neq 0$ $\sin x \neq 1$
   - (d) Write an equation for the line tangent to the graph of $f$ at $x = 0$.

3. Let $R$ be the region enclosed by the graphs of $y = (64x)^{\frac{1}{4}}$ and $y = x$. graph wat calc?
   - (a) Find the volume of the solid generated when region $R$ is revolved about the $x$-axis.
   - (b) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when region $R$ is revolved about the $y$-axis. ???

4. Let $f$ be the function given by $f(x) = 2\ln(x^2 + 3) - x$ with domain $-3 \leq x \leq 5$.
   - (a) Find the $x$-coordinate of each relative maximum point and each relative minimum point of $f$. Justify your answer.
   - (b) Find the $x$-coordinate of each inflection point of $f$.
   - (c) Find the absolute maximum value of $f(x)$. without calc? $2\ln(12+3) = ?$

GO ON TO THE NEXT PAGE

$\frac{dV}{dt} = 2$




5. The trough shown in the figure above is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time $t$, let $h$ be the depth and $V$ be the volume of water in the trough.

   (a) Find the volume of water in the trough when it is full.

   (b) What is the rate of change in $h$ at the instant when the trough is $\frac{1}{4}$ full by volume?

   (c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is $\frac{1}{4}$ full by volume?

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6. Let $f$ be a function such that $f(x) < 1$ and $f'(x) < 0$ for all $x$.

   (a) Suppose that $f(b) = 0$ and $a < b < c$. Write an expression involving integrals for the area of the region enclosed by the graph of $f$, the lines $x = a$ and $x = c$, and the $x$-axis.

   (b) Determine whether $g(x) = \dfrac{1}{f(x) - 1}$ is increasing or decreasing. Justify your answer.

   (c) Let $h$ be a differentiable function such that $h'(x) < 0$ for all $x$. Determine whether $F(x) = h(f(x))$ is increasing or decreasing. Justify your answer.

END OF EXAMINATION